

# A Novel Numerical Approach for the Analysis of 2D MEMS-Based Variable Capacitors Including the Effect of Arbitrary Motions

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## Abstract

A novel time-domain technique is proposed for the analysis of MEMS-based variable devices involving motion to arbitrary in-plane directions using the adaptive body fitted grid generation method with moving boundaries. MEMS technology is growing rapidly in the RF field and the accurate design of RF MEMS switches that can be used for phase shifting or reconfigurable tuners requires the computationally effective modeling of their transient and steady-state behavior including the accurate analysis of their time-dependent moving boundaries. Due to the limitations of the conventional time-domain numerical techniques, it is tedious to simulate these problems numerically. The new technique proposed in this paper is based on the finite-difference time-domain method with an adaptive implementation of grid generation. Employing this transformation, it is possible to apply the grid generation technique to the analysis of geometries with time-changing boundary conditions. A variable capacitor that consists of two metal plates that can move to arbitrary in-plane directions plates is analyzed as a benchmark. The

numerical results expressing the relationship between the velocity of the plates and the capacitance are shown and the transient effect is accurately modeled.

## 1. Introduction

The accurate knowledge of the electromagnetic field variation for a moving or rotating body is very important for the realization of new optical or microwave devices, such as the RF-MEMS structures used in phase-shifters, couplers or filters [1,2]. Computational method for moving boundary problems have been presented earlier in heat and fluid flow area[3]-[6]. In this paper, we propose a new numerical approach for the analysis of this type of problems that alleviates the limitations of the conventional time-domain techniques in the electromagnetic field[7]-[12] and shows good agreement with analytical results [13]. Employing the transformation with the time factor, it is possible to apply the grid generation technique of [14] to the time-domain analysis of geometries with moving objects. With such a grid, the FD-TD method can be solved very easily on a “static” (time-invariant)

rectangular mesh regardless of the shape and the motion of the physical region, something that makes it an especially good tool to analyze arbitrary shape and motion. In this paper, this simulation method is applied to the analysis of a two-dimensional MEMS variable capacitor with arbitrary in-plane motions of its interdigitated fingers.

## 2. General Theory of the Body-Fitted Grid Generation Method with Moving Boundaries

This technique is based on the finite-difference time-domain (FD-TD) method with an adaptive implementation of grid generation. The key feature of this method is that the time factor is added to the conventional numerical grid generation. We have improved the grid generation of [14] to the present one having a coordinate line coincident with arbitrarily shaped moving boundaries or moving bodies. Employing this transformation, it is possible to apply the grid generation technique to the analysis of geometries with time-changing boundary conditions. With such a grid, the FD-TD method can be solved very easily using a time-invariant square grid (rectangular computational region) regardless of the shape and the motion of the physical region. Employing the transformation with the time factor, the partial differential equation in the physical region  $(x, y, z, t)$  is related to the computational region  $(\xi, \eta, \zeta, \tau)$  as follows:

$$x = x(\xi, \eta, \zeta, \tau) \quad (1)$$

$$y = y(\xi, \eta, \zeta, \tau) \quad (2)$$

$$z = z(\xi, \eta, \zeta, \tau) \quad (3)$$

$$t = t(\xi, \eta, \zeta, \tau) \quad (4)$$

The inverse transformation is given by

$$\xi = \xi(x, y, z) \quad (5)$$

$$\eta = \eta(x, y, z) \quad (6)$$

$$\zeta = \zeta(x, y, z) \quad (7)$$

$$\tau = \tau(x, y, z) \quad (8)$$

According to the transformation, the first derivatives are transformed as follows,

$$\begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \\ \partial/\partial t \end{bmatrix} = K \begin{bmatrix} \partial/\partial \xi \\ \partial/\partial \eta \\ \partial/\partial \zeta \\ \partial/\partial \tau \end{bmatrix} \quad (9)$$

The inverse transformation is given by,

$$\begin{bmatrix} \partial/\partial \xi \\ \partial/\partial \eta \\ \partial/\partial \zeta \\ \partial/\partial \tau \end{bmatrix} = L \begin{bmatrix} \partial/\partial x \\ \partial/\partial y \\ \partial/\partial z \\ \partial/\partial t \end{bmatrix} \quad (10)$$

where the matrices  $K$  and  $L$  are given by

$$K = \begin{bmatrix} \partial \xi / \partial x & \partial \eta / \partial x & \partial \zeta / \partial x & \partial \tau / \partial x \\ \partial \xi / \partial y & \partial \eta / \partial y & \partial \zeta / \partial y & \partial \tau / \partial y \\ \partial \xi / \partial z & \partial \eta / \partial z & \partial \zeta / \partial z & \partial \tau / \partial z \\ \partial \xi / \partial t & \partial \eta / \partial t & \partial \zeta / \partial t & \partial \tau / \partial t \end{bmatrix} \quad (11)$$

and

$$L = K^{-1} = \begin{bmatrix} \partial x / \partial \xi & \partial y / \partial \xi & \partial z / \partial \xi & \partial t / \partial \xi \\ \partial x / \partial \eta & \partial y / \partial \eta & \partial z / \partial \eta & \partial t / \partial \eta \\ \partial x / \partial \zeta & \partial y / \partial \zeta & \partial z / \partial \zeta & \partial t / \partial \zeta \\ \partial x / \partial \tau & \partial y / \partial \tau & \partial z / \partial \tau & \partial t / \partial \tau \end{bmatrix} \quad (12)$$

By this transformation, there is an unique correspondence between the computational region and the physical region. The transformed region can be easily solved in the rectangular computational region by FD-TD method. The stability criterion for FD-TD algorithm is discussed in [8]

### 3. Two-Dimensional Variable Capacitor with Arbitrary Motions

The geometry to be considered here is shown in Fig.1. Under the combined effect of mechanical and electrical force, the two plates are assumed to move with different velocities to arbitrary in-plane directions. For the two-dimensional TM-propagation case, there are only  $E_x$ ,  $E_y$ ,  $H_z$  nonzero components with a time variation given by the following equations.

$$\frac{\partial H_z}{\partial t} = \frac{1}{\mu} \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x} \right) \quad (13)$$

$$\frac{\partial E_x}{\partial t} = \frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial y} - J_x \right) \quad (14)$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\varepsilon} \left( \frac{\partial H_z}{\partial x} + J_y \right) \quad (15)$$

where  $\varepsilon$ ,  $\mu$  are the constitutive parameters of the respective media. In Fig.1, the configurations of the physical and of the computational regions are shown. The interdigitated fingers are assumed to move to arbitrary directions in the  $xy$ -plane with velocities  $v$  and  $u$ , respectively and the direction of their motion is shown by the angles  $\theta_v$  and  $\theta_u$ . Using a coordinates' transformation technique, the time-changing physical region  $(x,y,t)$  can evolve to a time-invariant computational domain  $(\xi,\eta,\tau)$ . To transform the equations, it is easy to separate the physical region in 25 subregions, where  $(n,m)$  are the indices of each subregion in  $x$ - and  $y$ -direction. The number of subregions depends on the geometry of the moving parts of the geometry. Different subregions are characterized by different velocities in amplitude and/or direction. The transform equations between the physical and the computational regions are chosen as :

$$\xi = \alpha_{n-1} \frac{x - h_{n-1}(t)}{h_n(t) - h_{n-1}(t)} + \xi_{n-1} \quad (16)$$

$$\eta = \beta_{m-1} \frac{y - g_{m-1}(t)}{g_m(t) - g_{m-1}(t)} + \eta_{m-1} \quad (17)$$

$$\tau = t \quad (18)$$

,where  $n=1,2,3,4,5$   $m=1,2,3,4,5$  and  $h_0(t)$ ,  $h_1(t)$ ,  $h_2(t)$ ,  $h_3(t)$ ,  $h_4(t)$ ,  $h_5(t)$ ,  $g_0(t)$ ,  $g_1(t)$ ,  $g_2(t)$ ,  $g_3(t)$ ,  $g_4(t)$ ,  $g_5(t)$  are written in the following form assuming that the plate velocities remain constant for the whole time of their motion.  $\alpha_{n-1}$ ,  $\beta_{m-1}$  are coefficients to normalize the computational region. The coordinates  $x_1, x_2, x_3, x_4$ , and  $y_1, y_2, y_3, y_4$  represent the initial positions of the plates.

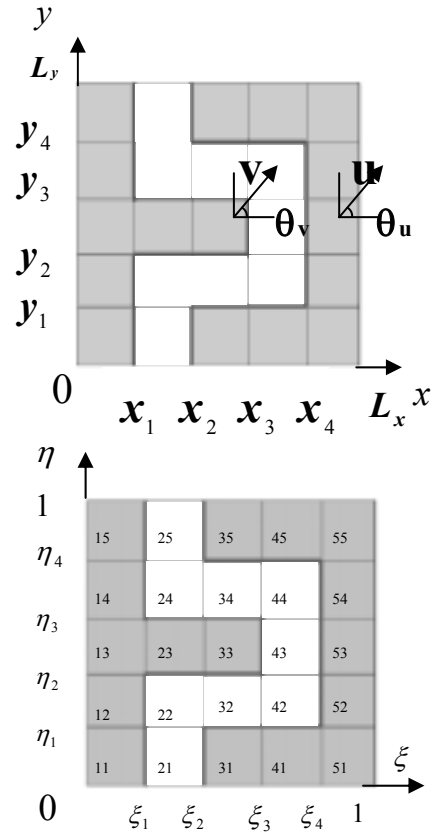


Fig1. Physical region (top graph) and computational (bottom graph) region

$$h_1(t) = x_1 + (v \cos \theta_v)t \quad (19)$$

$$h_2(t) = x_2 + (u \cos \theta_u)t \quad (20)$$

$$h_3(t) = x_3 + (v \cos \theta_v)t \quad (21)$$

$$h_4(t) = x_4 + (u \cos \theta_u)t \quad (22)$$

$$g_1(t) = y_1 + (u \sin \theta_u)t \quad (23)$$

$$g_2(t) = y_2 + (v \sin \theta_v)t \quad (24)$$

$$g_3(t) = y_3 + (v \sin \theta_v)t \quad (25)$$

$$g_4(t) = y_4 + (u \sin \theta_u)t \quad (26)$$

The functions  $h_1(t), h_2(t), h_3(t), h_4(t), g_1(t), g_2(t), g_3(t), g_4(t)$  describe the movement along the  $x$  and  $y$  axis, respectively, and allow for the realization of a rectangular grid with stationary boundary conditions, where  $h_0(t) = 0, h_5(t) = L_x, g_0(t) = 0, g_5(t) = L_y$ .

The partial time-derivatives in the transformed domain  $(\xi, \eta, \tau)$  can be expressed in terms of the partial derivatives of the original domain  $(x, y, t)$  using eqs.(16)-(26). The FDTD technique can provide the time-domain solution of the rectangular  $(\xi, \eta, \tau)$  grid. The stability criterion in this case is

chosen as  $c\Delta t \leq \delta/\sqrt{2}$ , where  $\delta = \Delta x_0 = \Delta y_0$ ,

assuming the grid is uniformly discretized in both directions. In general,  $\delta$  is a the minimum space increment (minimum cell size) for  $x$  and  $y$  directions[8].

### 3. Numerical Results

To validate this approach, numerical results are

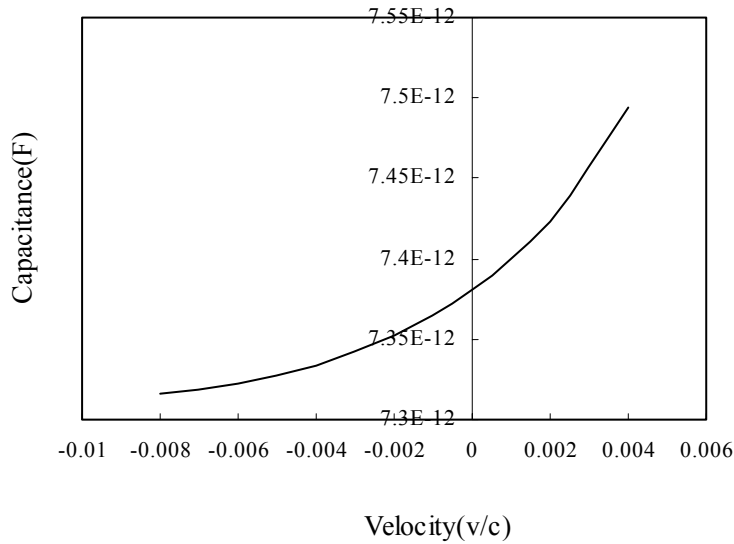


Fig2. Capacitance vs Velocity

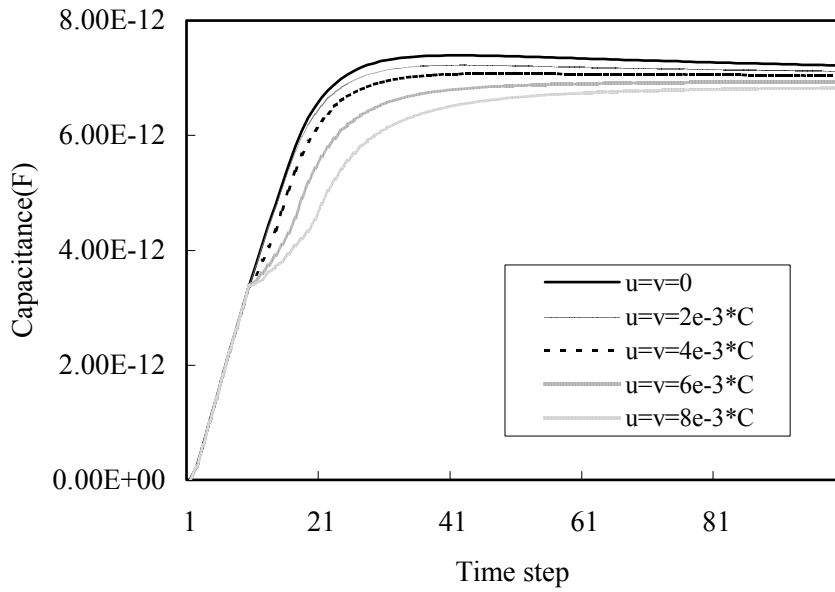


Fig.3 Time dependence of transient capacitance for each velocity, where plates go away from  $t=10$  time- step to  $t=60$  time- step

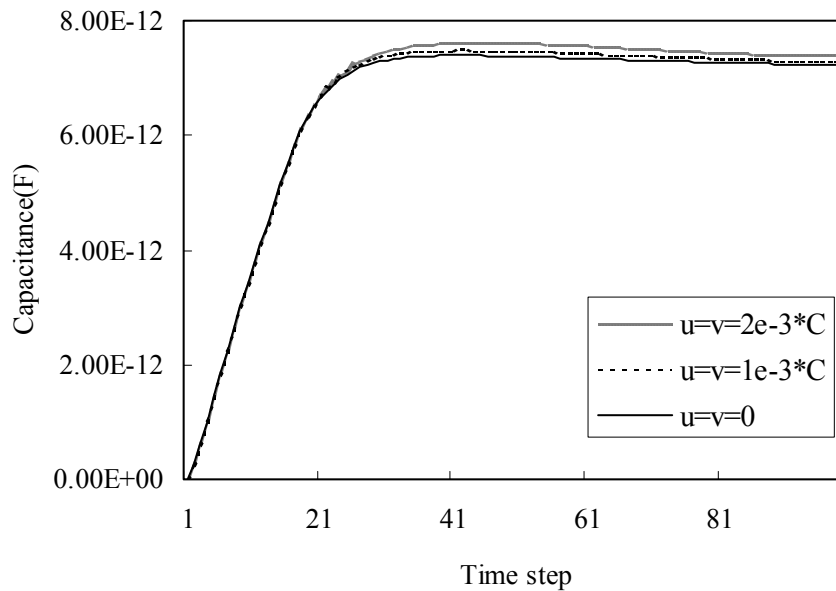


Fig.4 Time dependence of transient capacitance for each velocity, where plates approach each other from  $t=20$  time- step to  $t=60$  times- step

Calculated for a two-dimensional variable capacitor with its fingers moving only to the  $x$ -direction.

The grid includes  $200 \times 200$  cells and,  $L_x = L_y = L = 5\lambda$ ,  $\Delta x = \Delta y = L/200$ ,  $\Delta t = L/800c$ . In this case, as the plates are moving only to the  $x$ -direction away from each other, the angles are  $\theta_u = 0^\circ$ ,  $\theta_v = 180^\circ$  and as the plates are approaching each other, the angles are  $\theta_u = 180^\circ$ ,  $\theta_v = 0^\circ$ . The initial plate separation is  $L/5$  and the grid is terminated with Mur's absorbing boundary conditions. The relation between the velocity and the transient value of the capacitance between the moving fingers, assuming that they start to move away and approach each other at  $t=40$  time-step and stop at  $t=60$  time-step, is shown in Fig 2. The capacitance is derived from [15]. It can be observed that different velocity values lead to different values of the capacitance, since they affect the spacing of the fingers for a specific  $t_0$  time-step. Fig.3 displays computational results of the time dependence of the transient capacitance for velocity values in the range of  $u = v = 2 \times 10^{-3}c$  to  $u = v = 8 \times 10^{-3}c$ , assuming that the plates move away from each other from  $t=10$  time-step to  $t=60$  time-step. The horizontal axis indicates the normalized time expressed in time steps and the vertical axis indicates the value of the transient capacitance. The stationary value ( $v=u=0$ ) is displayed for reference reasons and demonstrates a (smoother) time-variation due to the time evolution of the excitation function itself. In Fig.4, the time dependence of the transient capacitance is demonstrated for various velocity values, assuming that the plates approach each other from  $t=20$  time-step to  $t=60$  time-step. Following this approach for the whole period of the motion of the fingers, it is easy to perform an accurate analysis of the transient response of the structure and predict the

ringing parasitic effects. It is clear that the transient effect is more pronounced for the higher values of velocity.

## Conclusions

A novel time-domain modeling technique that has the capability to accurately simulate the transient effect of variable capacitors with arbitrary in-plane motion of their plates has been proposed. This technique is a combination of the FDTD method and the body fitted grid generation technique. The key point of this approach is the enhancement of a space and a time transformation factor that leads to the development of a time-invariant numerical grid. The numerical results of the relation between the capacitance and the velocity of the motion are shown for a MEMS capacitor and demonstrate the proposed technique's unique computational advantages in the modeling of microwave devices with moving boundaries.

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