A Composite-Cell Multiresolution Time-Domain Technique for Design of Electromagnetic Band-Gap and Via-Array Structures

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Abstract **— In this paper the Haar-wavelet multiresolution time-domain (MRTD) scheme is modified in a way that enables the modeling of arbitrary positioned metals within a cell, leading to the development of composite cells that are useful for the simulation of highly detailed structures. The application of this technique to one such structure, an electromagnetic band-gap (EBG) resonator, is presented. The technique demonstrates a time-domain approach in which MRTD can be used to drastically reduce the number of cells needed to simulate a complex device while taking full advantage of the technique's inherent time- and space-adaptive gridding.**

I. INTRODUCTION

Advances in device processing are enabling the development of increasingly compact microwave circuits. These circuits incorporate a high degree of functionality through the combination of many microwave components in close proximity. These advanced devices often utilize geometries with high aspect ratios, small feature size, and moving parts. These characteristics, which are necessary to the operation of these devices, often lead to difficulties in predicting performance.

The simulation of these complex devices requires the use of extremely small elements or cells, which can tax many simulation tools beyond their limits. This has led to the use of a combination of methods, such as full-wave simulation and microwave circuit simulation, or, if higher accuracy is required, the use of a parallel full-wave simulator on specialized hardware. In order to simulate these complex devices in less time, methods which are more efficient without reducing accuracy are necessary.

The multiresolution time-domain (MRTD) [1] technique uses a wavelet discretization of Maxwell's equations to provide a time- and space- adaptive electromagnetic modeling scheme. The advantage of this method is that it can use much larger cells than similar full-wave time domain methods, such as finite-difference time-domain (FDTD) [2]. The number of basis functions used in each cell can be varied as a function of space and time. In this way, grids of complex structures can use high resolution cells in areas of large field variation, and lower resolution cells elsewhere. One such complex structure is an electromagnetic band-gap (EBG) resonator, such as that presented in Fig. 1 [3].

The EBG resonator shown here is designed to be compatible with modern printed circuit board technologies. A similar design can be used in any modern multilayer process, such as ceramic and organic substrates commonly used in building system-on-package modules. The structure uses a resonating chamber built using an arrangement of vias instead of metal walls. The via layout in this structure leads to complex grids in both the FDTD and MRTD techniques. A method that allows metals to intersect a cell in the MRTD grid would enable the use of larger MRTD cells, and thus increase the efficiency of the simulation.

In this paper a method is presented which enables the modeling of arbitrarily positioned conductors that intersect the MRTD grid. Combined with a method for modeling dielectric discontinuities within a cell [4], this technique can be used to create composite cells; cells that contain several conductor and dielectric interfaces. Both the theoretical formulation of the method and a demonstration of the application of the method to an EBG structure are presented. Another application of this technique is the modeling of a metal at any point in the grid. This is useful for a moving metal, as the grid does not have to be completely reformulated for each position of the metal. Instead, the MRTD resolution can be varied to allow the metal to be located wherever necessary. One important application of this technique is to RF-MEMS devices.

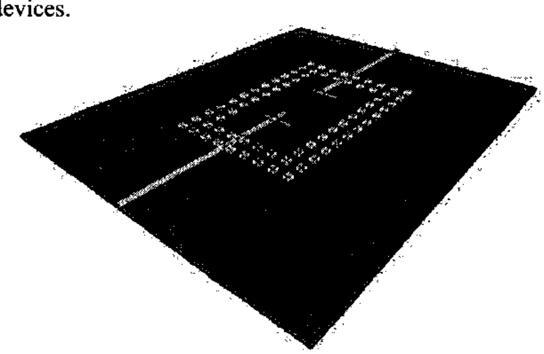

Fig. 1. EBG resonator, microstrip feed, cavity constructed using vias

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II. MRTD

The multiresolution time-domain technique draws its name from the application of multiresolution principles to Maxwell's equations. In the application of the method, the electric and magnetic fields are expanded into scaling and wavelet functions and then inserted into Maxwell's equations. The method of moments is then applied to these equations, leading to a time-marching scheme much like the finite-difference time-domain technique. The advantage of this technique over other methods is that wavelets can be added or subtracted during to the simulation at any point in the grid. In this way the grid can react to both complex geometry and rapid changes in the field as it propagates through the grid.

The choice of wavelet basis functions determines the characteristics of the MRTD scheme. In order to create an efficient scheme, Wavelet systems are usually chosen to create sparse discretizations of the modeled equations. In this analysis Haar wavelets (Figs. 2 and 3) will be used in space, Haar scaling functions only will be used in time [4]. Haar wavelets do not lead to as efficient of a scheme as other possible choices, however their finite domain nature enables the modeling of hard boundaries naturally, as well as limits the interaction of each cell to its nearest neighbors.

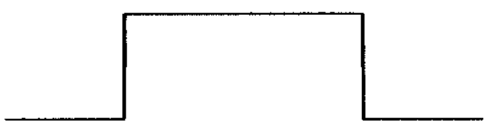

Fig. 2. Haar scaling function, φ

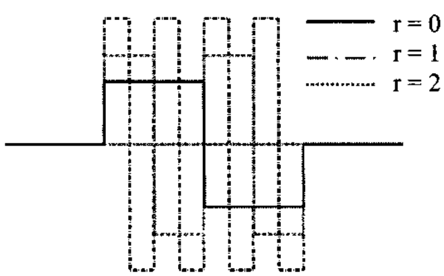

Fig. 3. Haar wavelets, ψ^r,r=0,1,2

In the following sections a method is presented which allows for the modeling of metals that are positioned within a cell. For simplicity, as well as space requirements, the derivations presented in this paper are in one dimension. The equations modeled are:

$$\frac{\partial}{\partial t}E_z(x,t) = \frac{1}{\varepsilon}\frac{\partial}{\partial x}H_y(x,t) \tag{1}$$

$$\frac{\partial}{\partial t}H_y(x,t) = \frac{1}{\mu}\frac{\partial}{\partial x}E_z(x,t) \tag{2}$$

When expanded into scaling and wavelet functions, the following expressions for E and H are found:

$$E_z(x) = \sum_{n,m} h_{n(t)}\left[\begin{array}{l} {}_nE_m^{z,\phi}\varphi_m(x) \\ +\sum_r\sum_p {}_nE^{z,\psi}_{m,r,p}\psi^r_{m,p}\end{array}\right] \tag{3}$$

$$H_y(x) = \sum_{n',m'} h_{n'}(t)\left[\begin{array}{l} H_{m'}^{y,\phi}\varphi_{m'}(x) \\ +\sum_r\sum_p {}_{n'}H^{y,\psi}_{m',r,p}\psi^r_{m',p}\end{array}\right] \tag{4}$$

In the above equations, $\varphi_m(x) = \varphi(x/\Delta x - m)$ and $\psi^r_{m,p} = 2^{r/2}\psi(2^r(x/\Delta x - m) - p)$, represent the scaled and translated versions of the scaling and wavelet functions. The positions of the scaling and wavelet functions are referred to by the parameter m for the E field and m' for the H field. It has been shown [5,6] that the relationship between m and m':

$$m' = m + \frac{1}{2^{r_{max}+2}} \tag{5}$$

leads to a doubling of resolution for each increased level of wavelet resolution.

When (3) and (4) are inserted into (1) and (2), and the method of moments is applied, individual update equations for each φ and ψ component are derived. The update equations for the E and H scaling functions are:

$$\begin{array}{l} {}_{n'+1}H^{y,\phi}_{m'} = {}_{n'}H^{y,\phi}_{m'} + \\ \dfrac{\Delta t}{\mu\Delta x}\left(\begin{array}{l} {}_nE^{z,\phi}_{m+1} - {}_nE^{z,\phi}_m + \\ \sum\limits_{r=0}^{r\max} 2^{r/2}({}_nE^{z,\psi}_{m+1,r,0} - {}_nE^{z,\psi}_{m,r,0})\end{array}\right)\end{array} \tag{6}$$

$$\begin{array}{l} {}_{n+1}E^{z,\phi}_{m} = {}_{n}E^{z,\phi}_{m} + \\ \dfrac{\Delta t}{\varepsilon\Delta x}\left(\begin{array}{l} {}_{n'}H^{y,\phi}_{m'} - {}_{n'}H^{y,\phi}_{m'-1} + \\ \sum\limits_{r=0}^{r\max} 2^{r/2}({}_{n'}H^{y,\psi}_{m'-1,r,2^r-1} - {}_{n'}H^{y,\psi}_{m',r,2^r-1})\end{array}\right)\end{array} \tag{7}$$

III. MRTD Intracell Metal Modeling

When inserting a PEC into an FDTD or MRTD grid, the boundary condition that must be enforced is that electric fields tangential to the PEC must be set to zero. This is a natural condition in FDTD, as metals can be placed along cells that coincide with the electric field locations in the Yee cell. This condition can be exactly duplicated in Haar MRTD by placing metals along the electric field locations in the modified Yee cell that represents the MRTD grid. If a metal only covers a portion of the cell, only the scaling and wavelet functions that intersect the metal need to be zeroed. By increasing the resolution, a metal intersecting any part of the grid can be represented. An example of this is presented in Fig. 4. In this case the metal splits a cell in two. The scaling function, 0^{th} resolution wavelet, and the first 1^{st} and 2^{nd} resolution wavelets intersect the PEC. The coefficients for these wavelets are zeroed. Higher resolution wavelets can be updated as in a normal MRTD grid. In this manner, the metal occupies the domain of only one of the highest resolution wavelets. The metal can be placed arbitrarily by selecting an appropriate wavelet resolution. If the position of the metal changes during simulation, the resolution of the cells that intersect the metal can be modified until a wavelet boundary exists on the metal (within an acceptable tolerance), and the lower resolution wavelets can be zeroed as before.

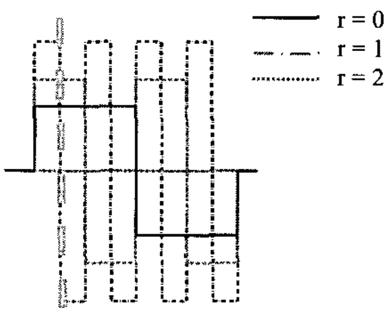

Fig. 4. PEC intersecting MRTD cell

Fig. 5 shows the results of a 1D Haar MRTD simulator with an intracell metal. This simulation uses an r_{max} of 1. The three graphs represent the electric field at different times. The first plot shows an initial time when pulses are traveling towards a PEC boundary. The second plot shows a later time when the pulse on the right is reflecting from the PEC. In the third plot, the right hand pulse is traveling in the opposite of its initial direction, while the left pulse is finishing reflecting from the PEC. The exploded view of the grid shows that half of the cell that contains the PEC has its coefficients set to zero (because rmax =1) while the r=1 wavelet function on the right is updated as normal.

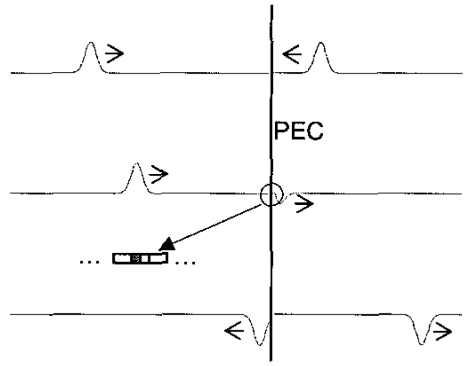

Fig. 5. Time domain plot of 1D-Haar MRTD simulation with intracell PEC

IV. EBG Resonator

EBG components use periodic arrangements of metals and dielectrics to create structures that only allow specific modes to propagate. One such structure is the EBG resonator in Fig. 1 [3]. This structure is similar to a solid wall resonator, however, it is compatible with multilayer processing techniques. The arrays of vias act as a metallic wall and thus create a resonating chamber. The feed and output microstrip lines are magnetically coupled to the cavity through a slot.

The S-parameters of this structure are presented in Fig. 6. This plot shows measured results, results from Ansoft's High Frequency Structure Simulator, and results from a parallel FDTD code. It can be seen that the FDTD results agree very well with the measured results, with the resonant frequency being almost exactly predicted. The only large discrepancy is the high frequency roll off, which is believed to be caused by fabrication error.

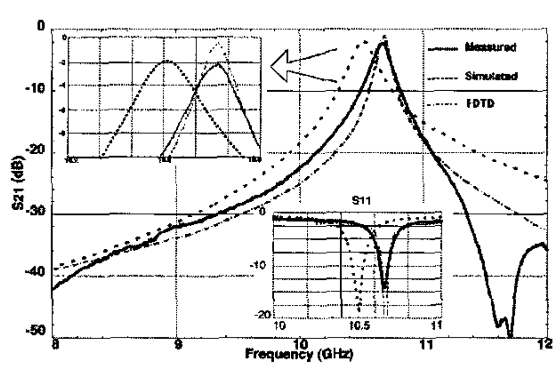

Fig. 6. Measured and simulated results of EBG resonator

In order to accurately model the resonator in FDTD, a very high resolution grid was required. Each via was modeled with four cells in each dimension. In addition, the area between the vias also required 4 cells. To match the grid to the physical structure, variable gridding [7] was used. The total number of cells in the grid was 2.56 million. In addition, the resonator required in excess of 250,000 time steps for the field decrease sufficiently for confidence in the results. On a 18 processor parallel Athlon MP 1800 cluster, the simulation took approximately 10 hours.

The FDTD modeling of this structure demonstrates the need for a high degree of accuracy and efficiency in a simulator for these devices. In the application of MRTD to the above structure, it is possible to reduce the number of cells in each direction by a factor of at least 4. This decreases the number of cells in the simulation at least 64 times. In the area of the vias, this is countered by the need for many wavelet resolution levels, however, it enables the use of low resolution cells away from the vias.

Using the subcell method presented in this paper, it is possible to further reduce the number of cells needed to simulate the structure. Due to the ability to place a metal within a cell, multiple vias can be modeled in a single MRTD cell. This is presented in Fig. 7. The MRTD cell represents several equivalent cells, each with its own coefficients. In this figure, the vias are represented by the shaded area. Because the vias are represented in the simulator as PECs, the coefficients of the scaling and wavelet functions intersecting the vias are set to zero. The coefficients in the remainder of the cell are updated as in a standard MRTD scheme. This allows significantly larger structures to be modeled, because while the resolution in the cells containing vias must be high, it can be lowered when the excitation is away from the via area, as the higher order wavelets can be ignored.

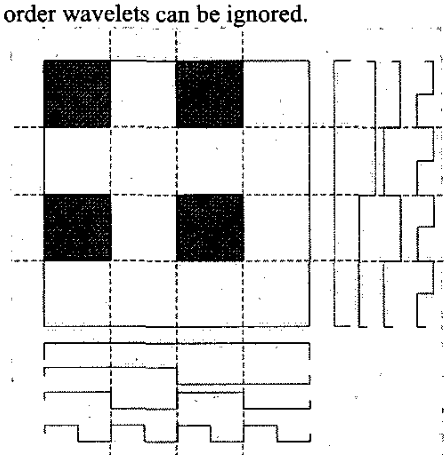

Fig. 7. 2D view of MRTD cell intersecting via holes

V. Conclusion

A method which enables the modeling of metals that intersect a multiresolution time-domain grid has been presented. The technique has advantages in that it enables the computationally efficient modeling of complex static structures as well as allows moving structures to be modeled in a natural way. This is done through the use of a composite cell, which can have several intracell dielectric and conductor discontinuities. The benefit of this technique was shown through the example of an EBG resonator. This structure exemplifies many real structures that exist in modern microwave circuits that require large computational resources, or the use of approximation techniques, in their simulation. Using this method the required computational resources can be greatly reduced.

Acknowledgement

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